

NAG Toolbox for MATLAB

e02bd

1 Purpose

e02bd computes the definite integral of a cubic spline from its B-spline representation.

2 Syntax

```
[dint, ifail] = e02bd(lamda, c, 'ncap7', ncap7)
```

3 Description

e02bd computes the definite integral of the cubic spline $s(x)$ between the limits $x = a$ and $x = b$, where a and b are respectively the lower and upper limits of the range over which $s(x)$ is defined. It is assumed that $s(x)$ is represented in terms of its B-spline coefficients c_i , for $i = 1, 2, \dots, \bar{n} + 3$ and (augmented) ordered knot set λ_i , for $i = 1, 2, \dots, \bar{n} + 7$, with $\lambda_i = a$, for $i = 1, 2, 3, 4$ and $\lambda_i = b$, for $i = \bar{n} + 4, \bar{n} + 5, \bar{n} + 6, \bar{n} + 7$, (see e02ba), i.e.,

$$s(x) = \sum_{i=1}^q c_i N_i(x).$$

Here $q = \bar{n} + 3$, \bar{n} is the number of intervals of the spline and $N_i(x)$ denotes the normalized B-spline of degree 3 (order 4) defined upon the knots $\lambda_i, \lambda_{i+1}, \dots, \lambda_{i+4}$.

The method employed uses the formula given in Section 3 of Cox 1975a.

e02bd can be used to determine the definite integrals of cubic spline fits and interpolants produced by e02ba.

4 References

Cox M G 1975a An algorithm for spline interpolation *J. Inst. Math. Appl.* **15** 95–108

5 Parameters

5.1 Compulsory Input Parameters

1: **lamda(ncap7)** – double array

lamda(j) must be set to the value of the j th member of the complete set of knots, λ_j for $j = 1, 2, \dots, \bar{n} + 7$.

Constraint: the **lamda(j)** must be in nondecreasing order with **lamda(ncap7 – 3) > lamda(4)** and satisfy
 $\mathbf{lamda}(1) = \mathbf{lamda}(2) = \mathbf{lamda}(3) = \mathbf{lamda}(4)$ and
 $\mathbf{lamda}(\mathbf{ncap7} - 3) = \mathbf{lamda}(\mathbf{ncap7} - 2) = \mathbf{lamda}(\mathbf{ncap7} - 1) = \mathbf{lamda}(\mathbf{ncap7})$.

2: **c(ncap7)** – double array

The coefficient c_i of the B-spline $N_i(x)$, for $i = 1, 2, \dots, \bar{n} + 3$. The remaining elements of the array are not used.

5.2 Optional Input Parameters

1: **ncap7** – int32 scalar

Default: The dimension of the arrays **lamda**, **c**. (An error is raised if these dimensions are not equal.)

$\bar{n} + 7$, where \bar{n} is the number of intervals of the spline (which is one greater than the number of interior knots, i.e., the knots strictly within the range a to b) over which the spline is defined.

Constraint: **ncap7** ≥ 8 .

5.3 Input Parameters Omitted from the MATLAB Interface

None.

5.4 Output Parameters

1: **dint** – double scalar

The value of the definite integral of $s(x)$ between the limits $x = a$ and $x = b$, where $a = \lambda_4$ and $b = \lambda_{\bar{n}+4}$.

2: **ifail** – int32 scalar

0 unless the function detects an error (see Section 6).

6 Error Indicators and Warnings

Errors or warnings detected by the function:

ifail = 1

ncap7 < 8 , i.e., the number of intervals is not positive.

ifail = 2

At least one of the following restrictions on the knots is violated:

$$\text{lamda}(\text{ncap7} - 3) > \text{lamda}(4) ,$$

$$\text{lamda}(j) \geq \text{lamda}(j - 1),$$

for $j = 2, 3, \dots, \text{ncap7}$, with equality in the cases $j = 2, 3, 4, \text{ncap7} - 2, \text{ncap7} - 1$, and **ncap7**.

7 Accuracy

The rounding errors are such that the computed value of the integral is exact for a slightly perturbed set of B-spline coefficients c_i differing in a relative sense from those supplied by no more than $2.2 \times (\bar{n} + 3) \times \text{machine precision}$.

8 Further Comments

The time taken is approximately proportional to $\bar{n} + 7$.

9 Example

```
lamda = [0;
         0;
         0;
         0;
         1;
         3;
         3;
         3;
         4;
         4;
         6;
         6;
```

```
        6;  
        6];  
c = [10;  
     12;  
     13;  
     15;  
     22;  
     26;  
     24;  
     18;  
     14;  
     12;  
     0;  
     0;  
     0;  
     0];  
[dint, ifail] = e02bd(lamda, c)  
  
dint =  
    100  
ifail =  
      0
```
